11986. Proposed by Martin Lukarevski, Goce Delčev University, Stip, Macedonia.

Let $x, y,$ and z be positive real numbers. Prove

$$4(xy+yz+zx) \le \left(\sqrt{x+y}+\sqrt{y+z}+\sqrt{z+x}\right)\sqrt{(x+y)(y+z)(z+x)}.$$

Solution by Arkady Alt , San Jose ,California, USA.

Let $a := \sqrt{y+z}, b := \sqrt{z+x}, c := \sqrt{x+y}$. Since numbers a,b,c are sidelengths of an acute triangle (because $a,b,c > 0,\ a^2+b^2 > c^2, b^2+c^2 > a^2, c^2+a^2 > b^2$) with area F, circumradius R, inradius r and semiperimeter s then inequality of the problem becomes $4\sum \frac{b^2+c^2-a^2}{2} \cdot \frac{c^2+a^2-b^2}{2} \le (a+b+c)abc \iff$
$2a^2b^2+2b^2c^2+2c^2a^2-a^4-b^4-c^4 \le 2s \cdot 4F \cdot R \iff 16F^2 \le 8s \cdot F \cdot R \iff$
$2F \le sR \iff 2rs \le sR \iff 2r \le R$ (Euler's Inequality)